**Progressive Education Society's**
**Modern College of Arts, Science & Commerce Ganeshkhind, Pune – 16**
**(Autonomous)**
**End Semester Examination – March / April 2024**
**Faculty: Science and Technology**

**Program : B.Sc. Code (BScGen03)** | **Semester : IV**
**Program (Specific) : Mathematics** | **Course Type: Core**
**Class : S.Y. B. Sc. (Regular)** | **Max. Marks : 35**
**Name of the Course : Linear Algebra** | **Course Code : 23-MT-241**
**Time : 2 Hrs.** | **Set B**

**Instructions to the candidate:**

1) ***All Sections are compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Draw a well labelled diagram wherever necessary.***

## SECTION-A

Q.1 Attempt any FIVE of the following. [5 x 2 = 10 Marks]

i) Give two examples of matrices which are in the row echelon form.

ii) Let V be a vector space. If u is a vector in V and k is a scalar such that $k \cdot u = 0$ then show that either $k = 0$ or $u = 0$.

iii) Define Basis of a vector space.

iv) Find the coordinate vector of $w = (2 , 5 , -3)$ relative to the basis $e_1 = (1 , 0 , 0)$ , $e_2 = (0 , 1 , 0)$ , $e_3 = (0 , 0 , 1)$ of $\mathbb{R}^3$

v) Determine basis and dimension of the subspace W of $\mathbb{R}^3$, where $W = \{ (x , y , z) \in \mathbb{R}^3 / x + y + z = 0 \}$.

vi) State Dimension Theorem for Linear Transformation

vii) Let V , W be two vector spaces and let $T : V \to W$ be a linear transformation. Then prove that $T( - u) = - T(u)$ , for all u in V

## SECTION-B

Q.2 Attempt any THREE of the following. [3 x 5 = 15 Marks]

i) Solve the following system of linear equations by Gauss Jordan

Elimination method $x + y + z = 9$ , $2x - 3y + 4z = 13$ , $3x + 4y + 5z = 40$.

ii) Show that the set of vectors S = { (1 , 2) , (1 , 0)} is a basis of $\mathbb{R}^2$.

iii) Prove that any two bases of a finite dimensional vector space V has same number of elements.

iv) Show that T : $\mathbb{R}^2 \to \mathbb{R}^2$ given by T(x , y) = (2x , y) is linear a transformation.

v) Let T : $\mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

T(x , y , z) = (x + y - z , x - 2y + z , - 2x - 2y + 2z).

Which of the following vectors are in ker (T) ?

(a) u = (1 , 2 , 3) (b) v = (1 , 2 , 1) (c) w = (-1 , 1 , 2).

## SECTION-C

Q.3 Attempt any TWO of the following. [2 x 5 = 10 Marks]

i) Reduce the following matrix A into reduced row echelon form , where

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 1 & 3 \\ 5 & 3 & 2 \\ 2 & 0 & 2 \end{bmatrix}.$$

ii) Show that W = { (x , y , z) $\in \mathbb{R}^3$ / x = 4y + z } is a subspace of $\mathbb{R}^3$.

iii) Let T : V → W be a linear transformation. Then prove that

(a)The kernel of T is a subspace of V.

(b) The range of T is a subspace of W.